The monitoring of the optics quality of DIMM instrument

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1 Introduction

A quantitative theory of measurement of optical turbulence (OT) with Differential Image Motion Monitor (DIMM) is based on certain priori assumptions [1]. One of these preconditions is the assumption of an ideal optical system, which builds two images of the same star, the differential displacement of which is measured. In the paper [2] it was shown that the presence of notable aberrations in real optical system can lead to systematic errors in the measured OT. The situation is exacerbated by the fact that commonly used as feeding optics the amateur class telescopes are not stable with respect to its optical quality. In many cases, focusing leads to the appearance of secondary coma, manifested in the DIMM images as astigmatism.

Another factor affecting the quality of the measured image is a registration by CCD camera with a non-zero exposure time. Wind vibration of the telescope significantly stretchs the image even at exposures of about 5 – 10 ms. If the registration is performed by frame transfer CCD, then each image is additionally burdened with weak vertical trace.

All these effects lead to the fact that actual images are not always suitable for accurate measurements. Some programs operating the DIMM, have control algorithm of the current quality of the images. Frames that didn't pass such control are ignored. Such an approach in real-time algorithm inevitably leads to the selection of measurements and greatly distorts the result.

Further analysis uses the data of measurement of OT with the MASS/DIMM instrument installed in 2007 on Mt. Shatdzhatmaz [1]. Total amount of data is 85 000 1-minute measurements obtained during the period 2007 – 2009. This work was done in preparation of these results to the re-processing and publication.

The measurements in the DIMM channel of the combined device are provided by the *dimm* program[2] which, apart from the basic data on the differential motion, calculates the additional features of the star images[3].

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[1] http://curl.sai.msu.ru/mass/download/doc/res2009eng.pdf

[2] http://curl.sai.msu.ru/mass/download/doc/infrasoft.pdf

[3] http://curl.sai.msu.ru/mass/download/doc/dimm_soft_description.pdf

2 Strehl's number evaluation by the *dimm* program

To estimate the Strehl's number of the actual image, the formula of theoretical dependence of the intensity in the center of the diffraction image on the total intensity F_0 [3] is used:

$$I_0 = F_0 \frac{\pi D^2 \Delta^2}{4\lambda^2 f^2}, \tag{1}$$

where D — the DIMM aperture diameter, f — the focal length of "telescope+DIMM" optical system and Δ — the size of CCD pixel. Replacing Δ/f by independently determined value – pixel scale M of CCD detector measured in arcsec per pixel, we have:

$$I_0 = F_0 \frac{\pi D^2}{4\lambda^2} \left(\frac{M}{206256}\right)^2 \tag{2}$$

It is assumed that the central intensity I_0 is a signal I_m at the maximum of the image. The measured ratio of the maximum signal for the total, divided by the ratio for the diffraction image of (2) gives the Strehl's nmber.

With the direct use of this formula there are several problems that lead to the incorrectness of the resulting estimate. The first problem arises from the fact that formula (2) is accurate if the $\Delta D \ll \lambda f$ or, equivalently, $I_0 \ll F_0$. Otherwise (pixel size is comparable to the size of the image peak), the maximum signal in the image will be noticeably lower then expected.

To minimize this error, the correction to the value of the maximum signal is calculated by the program *dimm* on the basis of position of the center of the image with respect to pixels and estimation of the radius of the image. For the image peak approximation the Gaussian profile is assumed. Our case is not so bad and the correction amounts to 10%.

The second problem relates to a quadratic dependence of maximum of intensity on wavelength λ while the real devices use whole of band sensitivity of the CCD detector. Calculations with a CCD ICX424AL detector response (industrial camera EC650 from Prosilica) show that for stars of spectral classes from B0V to K5III effective wavelength varies from 470 nm to 600 nm, what is equivalent to changing of the Strehl's number by 1.6 times.

Effective wavelength λ_{eff} is calculated by the distribution of energy $E(\lambda)$ and the reaction curve $Q(\lambda)$ as follows:

$$\frac{1}{\lambda_{eff}^2} = \frac{\int E(\lambda)Q(\lambda)\frac{1}{\lambda^2}d\lambda}{\int E(\lambda)Q(\lambda)d\lambda} \tag{3}$$

3 Preliminary analysis of the DIMM images Strehl's number

After correction of input data the cumulative distributions of the Strehl's number averaged over 1 min accumulation time were constructed. These curves are shown in Fig. 1 left. It is evident that the distribution for the left and right images are close to each other, differing only in the area of bad images ($S \approx 0.2$). Also, the distribution of mean values $S_m = (S_L + S_R)/2$ is shown.

Median of S_m throughout the whole period of the measurements is 0.41. In 13% of all cases the average Strehl ratio is less than 0.3. The right image is systematically worse than the left one.

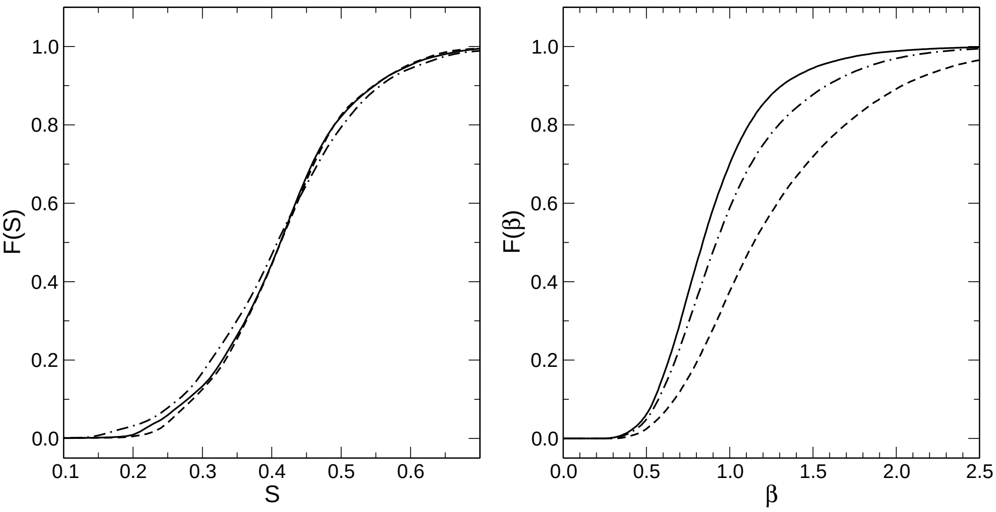

Figure 1: Left: distribution of Strehl's ratio for the left (dashed line) and right images (dot-dashed). Solid line shows the distribution of the mean number S_m. Right: cumulative distributions of seeing β for the three samples from the entire set of measurements. Solid line — $S_m > 0.4125$, dashed — $S_m < 0.4125$, dash-dotted line represents the distribution for $S_m > 0.3$

The picture on the right shows the cumulative distribution of seeing β for the three samples selected by the Strehl's number. Attention should be paid at strong dependence of these distributions from the selection rule. Thus, for the sample of $S_m > 0.41$ median β is $0.84''$ instead of $0.96''$ for the total population. Sample with $S_m < 0.41$ has the median value as large as $1.11''$. Even discarding marginal measurements with $S_m < 0.3$ leads to a noticeable shift in the median, which becomes $0.92''$.

The reason for this situation isn't that bad images lead to an overestimation of the OT, but on the contrary, strong turbulence causes degradation of the star image even for 10 cm aperture typical in DIMM device. This effect has been discussed in detail in [4] where a special comparison of two identical DIMM was performed in different alignment status.

4 OT effect in image distortion in the DIMM

To estimate the contribution of the optical turbulence (OT) in the DIMM images formation, an approximate expression can be obtained by assuming that exposure of frames is short and wavefront tilts don't affect it. We assume, as in the work [4], that the phase fluctuations of wavefront aberrations caused by the telescope and the atmosphere, are uncorrelated and small. Consequently, the total variance of the fluctuations can be written as the sum of the variances of fluctuations caused by the atmosphere of σ_a^2 and aberrations of the telescope and instrument optics σ_o^2:

$$\sigma^2 = \sigma_o^2 + \sigma_a^2, \tag{4}$$

what is equivalent to presentation of the Strehl's number S of image in focal plane of the instrument as a product of Strehl's number of the optical system S_o and turbulent atmosphere

S_a

$$S = S_o \, S_a \tag{5}$$

Thus, to evaluate the instrumental component, it is sufficient to divide measured estimation of image Strehl ratio by the contribution of the atmosphere S_a for a given OT power.

Atmospheric Strehl's number can be estimated using the relation for short-exposure image from Fried paper [5]:

$$\sigma_a^2 = \sigma_3^2 = 0.134 \Big(\frac{D}{r_0}\Big)^{5/3}, \tag{6}$$

however, this formula is accurate for case $D \gg r_0$ only, and because of it the authors [4] using this approach, introduced additional factor determined from measurements and computer modeling.

There is a more rigorous way to calculate the atmospheric Strehl ratio, based on its definition in terms of the optical transfer function OTF of the optical system:

$$S_a = \int_0^{f_c} T_a(f)\, df \Big/ \int_0^{f_c} T_0(f)\, df \tag{7}$$

where T_0 — diffraction limited OTF of the system, T_a — OTF with OT contribution and integration is up to the cutoff frequency $f_c = D/\lambda$.

It is known that the diffraction OTF is the autocorrelation function of the input aperture or Fourier Transform of the Airy function:

$$T_0(z) = \frac{2}{\pi} \Big[\arccos z - z\sqrt{1 - z^2} \Big] \tag{8}$$

where $z = f/f_c$ — dimensionless spatial frequency. The integral of this function throughout the frequency range:

$$\int_0^1 T_0(z) = \frac{4}{3\pi} \tag{9}$$

For the Kolmogorov's turbulence spectrum, expression for the OTF T_a was obtained in [6]. Moreover, in the case of long exposure (ie, with input from the tilts of the wavefront):

$$T_a^{LE}(z) = T_0(z) \exp\left(-3.44 \Big(\frac{D}{r_0}\Big)^{5/3} z^{5/3} \right), \tag{10}$$

and, respectively, in the case of short exposure, relevant in the case of DIMM:

$$T_a^{SE}(z) = T_0(z) \exp\left(-3.44 \Big(\frac{D}{r_0}\Big)^{5/3} z^{5/3} \Big[1 - (z)^{1/3}\Big] \right) \tag{11}$$

One can see that the integrals of these functions in whole frequency domain depend on the single parameter $s = (D/r_0)^{5/3}$. Calculated with formula (7) Strehl ratios S_a both for (10) and (11) are shown in Fig. 2 as function of this parameter.

There is a very accurate approximation of the behavior of atmospheric Strehl in range $r_0 > 0.25D$ in the form of

$$S_a = e^{-0.12125\, s + 0.00287\, s^2}. \tag{12}$$

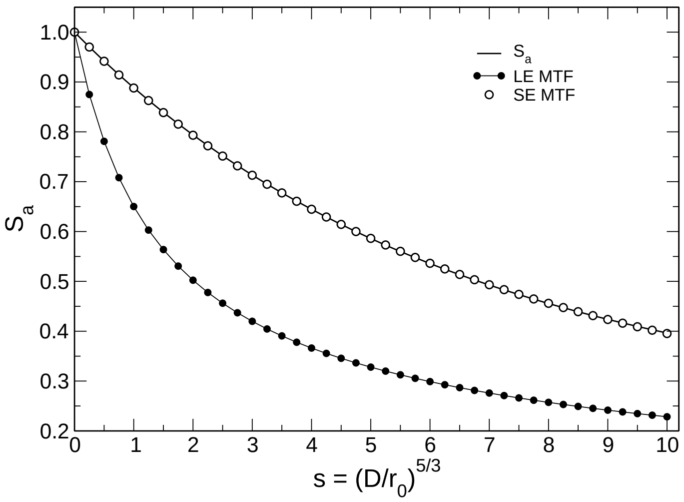

Figure 2: Dependence of the atmospheric Strehl ratio S_a on OT intensity. Long-exposure case (10) — black points. Short-exposure case (formula 11) — empty circles. The solid thick curve — the approximation of formula (11) by the relation (12).

It's clear that this approximation isn't dependend on D nor on λ and so it is applicable for any particular DIMM instrument. The value of s depends linearly on the OT intensity and can be expressed via the measured values of the following way:

$$s = \left(\frac{D}{r_0}\right)^{5/3} = \left(\frac{D}{\lambda}\right)^2 \cdot \frac{\sigma^2_{l,t}}{K_{l,t}}, \tag{13}$$

where $\sigma^2_{l,t}$ — the differential longitudinal and transversal motion in squared radians, and $K_{l,t}$ — the corresponding coefficient in the formula relating the $\sigma^2_{l,t}$ to the OT intensity at the line of sight. As λ one should use the effective wavelength for real CCD detector and the observed star, calculated earlier for the correct evaluation of the measured Strehl's ratio by formula (3).

5 Study of the optics quality of DIMM instrument

The dependence which shown in Fig. 2 demonstrates that the degradation of images due to atmospheric turbulence, in a typical situation (the seeing $\beta = 1''$, $D/r_0 \approx 1$) is not large, about 10% only. With OT growth, the degradation increases markedly, reaching 30% at $\beta = 2''$. Nevertheless, the contribution of atmospheric turbulence in the image degradation is proved to be sufficient to significantly alter the results presented in section 3.

The cumulative distribution of estimates of Strehl ratio after correction for the OT contribution (instrumental Strehl's number) is presented in Fig. 3 in left. The median of S_m increased to 0.47, while the median for left and right images are equal to 0.46 and 0.48, respectively. As before, difference in the quality of left and right images are noticeable especially in the region of low Strehl's. Fractions of marginal value $S < 0.3$ decreased to 3% and 7% for the left and right images.

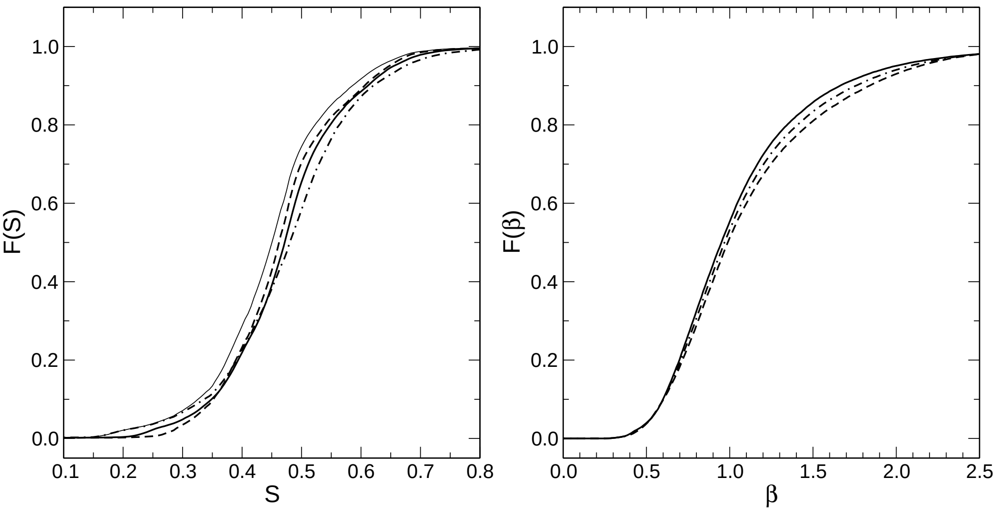

Figure 3: Left: cumulative distributions of the Strehl's number for the left (dashed line) and right images (dot-dashed line) after taking into account the OT contribution. The solid line represents the distribution of the average number S_m, thin solid — the distribution of $min(S_L, S_R)$. Right: cumulative distributions of the seeing for three samples. Solid line — sample $S_(min) > 0.45$, dashed — sample $S_(min) < 0.45$, the dot-dash line represents the overall seeing distribution

This region of the distribution is associated with a significant misalignment in the telescope in the period from March to July 2009 because of the tilt corrector plate with a secondary mirror. The fact is confirmed by the curve shown in Fig. 4. At that time there was a progressive divergence of optical axes of the finder and the device along a horizontal line connecting the DIMM apertures. To the alignment moment the shift reached a value $7 - 8'$. For an explanation of such shift the tilt of the secondary mirror around a vertical axis on $12 - 15'$ or $0.2 - 0.25°$ is required.

After taking into account the atmospheric contribution of S_a the effects of selection for the seeing was almost disappeared. In Fig. 3 on right, the cumulative distributions of β are presented. The distributions are constructed for the different samples of data. It is seen that the curves for the sample $S_{min} > 0.45$ (better than the median for a minimum of two of Strehl's number) and for the $S_{min} < 0.45$ are very close, a small difference is seen in the situation of large OT. Their medians are equal to $0.95''$ and $0.99''$ when the general median is equal to $0.97''$.

Table 1 shows more detailed information on specific points of distributions of the quality of images β at different thresholds selection of reliable measurements. It is seen that the maximum difference (for the last quartile) is $0.05''$ what is quite unimportant for large OT. In the case of weak OT (first quartile), the maximum difference due to selection criteria are much less and $\sim 0.01''$.

Of course, such small differences follows of the fact that the overwhelming data volume was obtained with the good quality of the optical system of the telescope and instrument. Monotonic decrease of quartiles with an increase in the threshold indicates that the theoretical description of (5 and 11) does not fully compensate the initial dependence of the total Strehl ratio on OT power. However, the residual dependence may be a consequence of the selection effect, so that

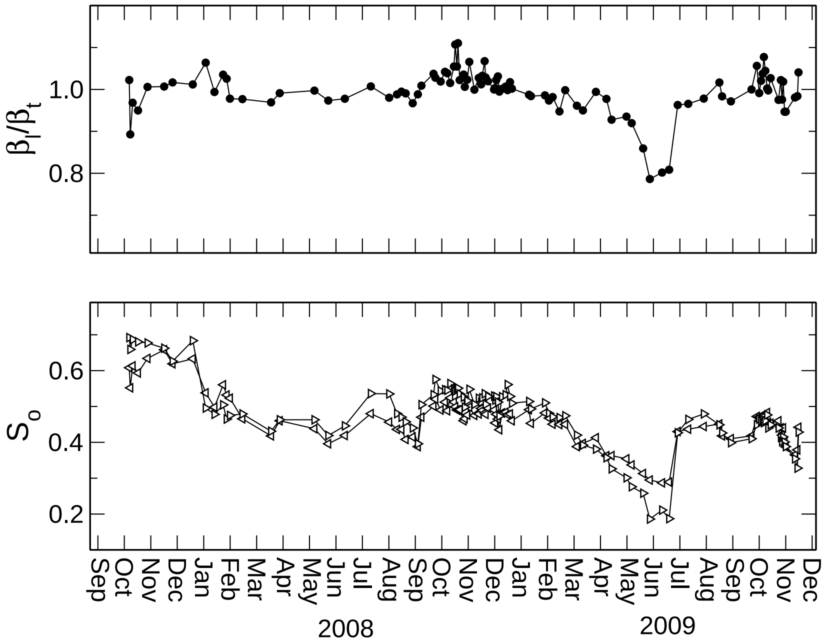

Figure 4: Top: evolution of the median ratio of the longitudinal and transverse motion during the period of observations. Bottom: behavior of the instrumental Strehl's number for the left (triangle to the left) and right (triangle to the right) DIMM images during the period of measurement

the maintenance of an additional correction factor seems superfluous.

The fact that the use of the criterion $S_{min} > 0.3$ is appropriate for rejection of measurement, is confirmed by the evolution of the ratio β_l/β_t over the whole period, which is shown in Fig. 4 at the top. It is evident that during the measurements with strongly misaligned telescope, this ratio is significantly different from 1, which is most likely due to the influence of the aberrations on the results.

6 The telescope vibration effect

The wind excites vibrations of the telescope at frequencies close to its natural frequencies in the range of 5 – 10 Hz. The amplitude (half span) of the Y vibrations across the DIMM base (in altitude) is about twice more than in the azimuth X. To estimate the magnitude of vibration, the rms value of displacement of the common center of two images over 1 – 2 s measurement was used. Even in the absence of wind, this value is not equal to 0, because there are in-phase motion caused by atmospheric turbulence and the displacement due to tracking and guiding. Actually wind vibrations starts to grow when the averaged wind speed exceeds 3 m/s (Fig. 5 left). Dependence of the total value of vibration $R = (X^2 + Y^2)^{1/2}$ is slightly higher than the curve for the vibration of Y.

It should be noted that the fraction of the observations made under considerable vibrations $R > 3''$ amounts only 15%, and $R > 6.4''$ only 3% of all cases. It is due to the fact that the

Table 1: The quartiles of the seeing β distributions under different selection criteria

criteria	25%	50%	75%
no selection	0.751	0.969	1.292
$S_{min} > 0.30$	0.747	0.965	1.296
$S_{min} > 0.35$	0.746	0.961	1.280
$S_{min} > 0.40$	0.746	0.959	1.265
$S_{min} > 0.45$	0.739	0.950	1.243
$S_{min} < 0.45$	0.765	0.989	1.347

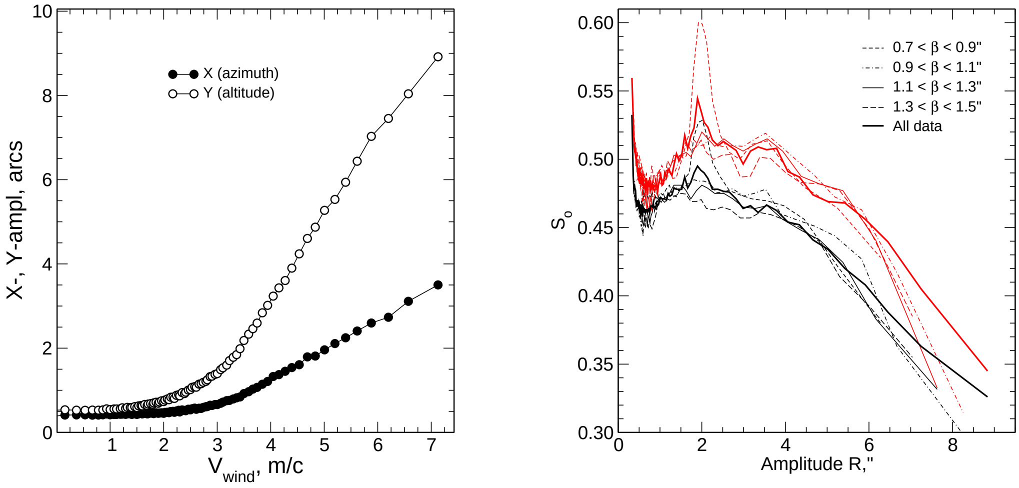

Figure 5: Left: the dependence of the rms value of vibrations of the telescope by azimuth (black dots) and by altitude (empty circles) on the ground wind speed. Right: The dependence of the median values of the Strehl (black lines for the left image, light — for the to right one) on the amplitude of vibration for different samples of OT power

median wind speed is only 2.3 m/s and measurements stops when the speed is more than 9 m/s.

Assuming that wind vibrations are close to harmonic, and knowing the frequency of these oscillations we can estimate that the images are stretched in the direction of the vibrations to the $\approx 1''$ for the vibrations of the amplitude $R \approx 5''$ and exposure of 4 ms, which leads to decrease in the maximum signal and reduces the Strehl's number for images.

This effect is observed indeed. In Fig. 5 right the dependencies of the median Strehl's numbers on the vibration amplitude are presented. To avoid the effect of uncompensated impact of OT power, these curves were constructed for four fairly narrow ranges of values of β — they all exhibit similar behavior. The behavior in range $R < 3''$ is not connected with the wind vibrations and is determined by other factors. We can conclude that the effect reduces the number of Strehl's number to about 0.35 when the amplitude $R \approx 8''$, however, due to the small amount of such measurements, they don't affect the overall picture.

However, due to interference with the stellar scintillation, almost independent in different

DIMM apertures, the increase of the differential motion power should be expected along the direction of vibrations — in our case the transversal components

Micro-accelerometer type ADXL203 from Analog Device Company having a sensitivity of better than 1 mgal ($\sim$ 1 cm/s^2) is quite suitable for independent information on the actual mechanical vibrations of the telescope. In the typical conditions of use of the instrument DIMM, such acceleration correspond to vibrations with an amplitude of about $1 - 2''$.

7 Conclusions

The results of analysis of data obtained with the MASS/DIMM instrument at Mt. Shatdzhatmaz during October 2007 to November 2009 confirm that the validation of the measurements by using the DIMM image Strehl's number should be used with great caution, because it leads to a selection effect which statistically belittles the atmospheric optical turbulence.

However, the quality control of the optical system is needed. Such control by using estimates of the Strehl ratio is quite possible after accounting the effect of OT, but certainly not for separate frames, because at that time information about the current OT power is non-existent. Culling non-reliable measurements can be made after the obtaining of complete picture of the current situation. In any case, the control should not lead to the loss of measurement data.

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