The PREAT description

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June 29, 2016

1 Usage

The code *preat* can be located anywhere on the disk. It uses the spectral reaction curves of the DIMM camera, `ccd.crv` and approximate spectral energy distributions of the observed stars. In the file structure of `dimm_tools` these data are located in the directories `./data/data/` and `./data/specra/`. The location can be changed; to do so, edit the following lines in the file `defnames.h`:

```
const std::string  program_spe = program_root + "data/spectra/";
const std::string  program_dat = program_root + "data/data/";
```

The input data for the program are taken from the standard input `stdin` and sends the results to the standard output `stdout`. It can be used in a variety of ways. For batch processing of the archival data, the command can look like

```
cat *.stm | ./preat > whole.dat
```

For processing the data in real time during observations, the command can be

```
tail -f -n 1000 yymmdd-dimm.stm | ./preat > yymmdd-dimm.pre
```

provided that *preat* is launched immediately after the start of the measurements. The option `-n 1000` is set in order not to miss the preamble of the current DIMM file.

2 Main formulae

The *preat* code serves for converting the output data of the program *dimm* into full intensity of optical turbulence (OT) J, in the near-field approximation,

$$J = \int C_n^2(h)\,dh, \tag{1}$$

where the integral is taken over full height of he atmosphere. The OT intensity is expressed in the units of $\mathrm{m}^{1/3}$.

We assume that the MASS/DIMM instrument and its camera are oriented in such way that the centers of the subapertures are located on the X axis and the two star images are also separated in the X direction.

The longitudinal (along the line joining the centers of the subapertures) and transverse dispersions, σ_l^2 and σ_t^2, as well as the full dispersion $\sigma_\mathrm{c}^2 = \sigma_\mathrm{l}^2 + \sigma_\mathrm{t}^2$ of the differential image motion are described by the formula that is easily derived from the standard relation containing the Fried parameter r_0 [1]

$$\sigma_\mathrm{l,t,c}^2 = 16.7 \cdot K_\mathrm{l,t,c}\, D^{-1/3} J, \tag{2}$$

The proportionality of the dispersion to the OT integral is obvious.

To simplify, we modified the sense of the coefficient $K_\mathrm{l,t,c}$, interpreting it as the coefficient relating the OT to the full dispersion of the image motion:

$$J = \frac{\sigma_\mathrm{l,t,c}^2}{K_\mathrm{l,t,c}}. \tag{3}$$

The coefficients $K_\mathrm{l,t,c}$ are computed from the formulae given in [1] which, for the G-tilt, are

$$
\begin{aligned}
K_{\rm t} &= 16.70 \cdot 0.358\,(D^{-1/3} - 0.811\,d^{-1/3}) \\
K_{\rm l} &= 16.70 \cdot 0.358\,(D^{-1/3} - 0.541\,d^{-1/3}) \\
K_{\rm c} &= K_{\rm t} + K_{\rm l}
\end{aligned}
\tag{4}
$$

and for the Z-tilt

$$
\begin{aligned}
K_{\rm t} &= 16.70 \cdot 0.364\,(D^{-1/3} - 0.798\,d^{-1/3}) \\
K_{\rm l} &= 16.70 \cdot 0.364\,(D^{-1/3} - 0.532\,d^{-1/3}) \\
K_{\rm c} &= K_{\rm t} + K_{\rm l}
\end{aligned}
\tag{5}
$$

The full coefficient was introduced in [2] because the correction of the full dispersion to the zero exposure depends on the wind direction much less than for the longitudinal and transverse dispersions.

3 The algorithm

For each full measurement cycle of `accumtime` (accumulation time) duration, the input to *preat* consists of $N =$ `accumtime/basetime` measurements of the mean distances between the two spots, $\bar{x}$, $\bar{y}$, dispersions of the differential image motion $\tilde{\sigma}_x^2$, $\tilde{\sigma}_y^2$, temporal covariances between adjacent exposures $\tilde{c}_x$, $\tilde{c}_y$, and the estimates of the coordinate noise $\tilde{r}_x^2$, $\tilde{r}_y^2$ computed from $n \approx 100 \div 200$ exposures taken during `basetime`.

Note: the files `*.stm` contain, instead of the dispersions, the rms values $\tilde{\sigma}_x$, $\tilde{\sigma}_y$ to facilitate quick estimates of the seeing $\beta \sim \sigma^{5/6}$. The coordinate noise is also listed as its rms value, $\tilde{r}_x$, $\tilde{r}_y$. Squares of these numbers are used in the following processing.

In the following, the image motion power calculated from the dispersions is called high-frequency power ($\nu > 1$ for `basetime` $=$ 1s), while the dispersion calculated from the variance of the average coordinates $\bar{x}$, $\bar{y}$ is called low-frequency power ($\nu < 1$). The calculation proceeds in the following steps.

Subtraction of the coordinate noise. The dispersions $\tilde{\sigma}_x^2$, $\tilde{\sigma}_y^2$ are biased by the coordinate noise, to be subtracted from each of the N measurements according to [1, 3] :

$$
\sigma_x^2 = \tilde{\sigma}_x^2 - \tilde{r}_x^2, \qquad \sigma_y^2 = \tilde{\sigma}_y^2 - \tilde{r}_y^2. \tag{6}
$$

Given the orientation of the instrument, $\sigma_{\rm l}^2 = \sigma_x^2$ and $\sigma_{\rm t}^2 = \sigma_y^2$. For the low-frequency image motion, the coordinate noise is $n^{1/2}$ less, so it is neglected.

Correction to zero exposure time. The method developed in [2] is used to correct the data for the finite exposure time. It uses the temporal covariance of the differential image motion $\tilde{c}_x$, $\tilde{c}_y$, which is relevant when the exposures are taken in rapid succession. When the exposure time is $\tau =$ `exposure` (typically 2.5 ms) and the period of the images (cadence) is $T =$ `basetime`$/n$ (typically 5 ms), there is a significant temporal covariance. We use the following relations:

$$
\alpha = \frac{3}{2}\,\frac{\tau^2}{\tau^2 + 6\,T^2}. \tag{7}
$$

Typically, $\alpha = 0.06$. For each component of the image motion,

$$\sigma_0^2 = \sigma^2 \frac{1 - \alpha\rho}{1 - \alpha} \tag{8}$$

where the correlation coefficient $\rho = \tilde{c}/\sigma^2$.

Calculation of the low-frequency image motion. The low-frequency part of the image motion $\hat{\sigma}^2$ is computed from the differences. This removes trends, which are usually caused by slow thermal deformations of the telescope optics and mechanics. The method is a common one. For example, for the x-component,

$$\hat{\sigma}_x^2 = \frac{1}{2(N-1)} \sum_{i=0}^{N-1} (\bar{x}_i - \bar{x}_{i+1})^2. \tag{9}$$

Naturally, no correction to the zero exposure is needed.

Conversion of dispersion from pixels to radians. To express in radians the differential image motion computed in pixels, the pixel scale `scale` (in acrseconds) is used, taken from the header of the processed file. The resulting conversion factor is $scale = 4.8481368 \cdot 10^{-06} \cdot \mathtt{scale}$ and all dispersions are scaled as $\sigma^2[\mathrm{rad}^2] = scale^2\,\sigma^2[\mathrm{pxs}^2]$.

Calculation of the turbulence integral J To compute the turbulence integral OT (lines with prefix “A”) using the formula (3), the high-frequency and low-frequency variances are summed to the full variance and the coefficients K calculated from (4) or (5) are used.

As noted in [1], the formulas for the Z-tilt (5) are a better match to the centroid estimates and, moreover, they are closer to the standard coefficients from [4], so we use the Z-tilt coefficients by default. The coefficients can be changed to those of G-tilt in the module `parameter_set.cpp`, and the program should be recompiled.

Finally, the OT is translated to the zenith by the trivial relation $J_0 = J_z/M(z)$, where $M(z)$ is the airmass of the program star.

Estimation of the Strehl ratio The image motion is measured in the wide spectral band of the CCD detector in DIMM, and, as a consequence, the effective wavelength λ_{eff} depends on the spectral type of the stars and the Strehl ratios show a dependence on the spectral type. The λ_{eff} varies by more then 25% for red and blue stars. Generalizing the formula (10) from [1], we can compute the expected ratio of the central intensity of the PSF I_0 to the total intensity F for the case of a polychromatic source:

$$\frac{I_0}{F} = \frac{\pi D^2\, scale^2}{4} \cdot \left(\frac{1}{\lambda^2}\right)_{\mathrm{eff}} \tag{10}$$

The last term is the inverse wavelength $1/\lambda^2$ weighted by the product of the detector spectral response $Q(\lambda)$ and the stellar spectrum $E(\lambda)$.

$$\left(\frac{1}{\lambda^2}\right)_{\mathrm{eff}} = \frac{\int E(\lambda) Q(\lambda) \frac{1}{\lambda^2} \mathrm{d}\lambda}{\int E(\lambda) Q(\lambda) \mathrm{d}\lambda} \tag{11}$$

In principle, the weight should include the atmospheric transmission. Its effect is small for the typical CCD response curve, but it can become non-negligible at large airmass for white stars.

The Strehl ratio is the ratio of the measured central intensity (in units of ful flux) to the theoretical formula for an ideal case (10). Image blurring caused by the seeing is accounted for by the approximate formula derived in the document "The monitoring of the optics quality of DIMM instrument"[1]

4 Format of the preat output

"F" string format

1. Prefix "F"
2. Date
3. Mean moment of the of accumulation (typical accumulation 1 min)
4. Airmass for this moment.
5. Total number of processed frames
6. Mean total flux in the left image
7. Relative RMS error of this mean
8. Mean scintillation index in the DIMM aperture for the left image
9. Mean total flux in the right image
10. Relative RMS error of this mean
11. Mean scintillation index in the DIMM aperture for the right image
12. Seeing-corrected Shtrel ratio for the left image
13. Seeing-corrected Shtrel ratio for the right image

"I" string format All parameters of the differential image motion are given in these lines in [pxs^2] or [pxs].

1. Prefix "I"
2. Date
3. Mean moment of accumulation
4. Mean longitudinal separation
5. Mean transverse separation
6. Low frequency longitudinal motion variance
7. Low frequency transverse motion variance
8. Averaged longitudinal motion variance (High frequency variance)
9. Averaged transverse motion variance (High frequency variance)
10. RMS error of the average longitudinal variance
11. RMS error of the average transverse variance
12. Averaged longitudinal correlation coefficient
13. Averaged transverse correlation coefficient

[1] `http://curl/mass/download/doc/shtrel_controls.pdf`

"A" string format. The values are turbulence intensity corrected for airmass and given in $m^{1/3}$. To convert into the seeing in arcseconds, use the factor $2.0 \times 10^7 \cdot J^{3/5}$

1. Prefix "A"
2. Date
3. Mean moment of accumulation
4. Full turbulence intensity calculated with longitudinal motion
5. Full turbulence intensity calculated with transverse motion
6. Full turbulence intensity calculated with common motion
7. Low frequency turbulence intensity calculated with longitudinal motion
8. Low frequency turbulence intensity calculated with transverse motion
9. Low frequency turbulence intensity calculated with common motion

Bibliography

[1] A. Tokovinin. From Differential Image Motion to Seeing. *PASP*, 114:1156–1166, October 2002.

[2] V. Kornilov and B. Safonov. Differential image motion in the short-exposure regime. *MNRAS*, 418(3):1878–1888, December 2011.

[3] A. Tokovinin and V. Kornilov. Accurate seeing measurements with MASS and DIMM. *MNRAS*, 381:1179–1189, November 2007.

[4] M. Sarazin and F. Roddier. The ESO differential image motion monitor. *A&Ap*, 227:294–300, January 1990.